EXTENDED ESSAY

MATHEMATICS

HIGHER LEVEL

**My Pension Plan**

***If I put a specific amount of money in euros into a savings account each year while I am working, how much additional money per year throughout my retirement period can I expect to receive?***

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**ABSTRACT**

The public debate on sustainability of contemporary public pension systems encouraged me to reflect on my future and to write this essay. I decided to find a way of ensuring a dignified standard of living in old age in the framework of private pension system. My main goal was to answer the following research question: *If I put a specific amount of money in euros (€1,200) into a savings account each year while I am working, how much additional money per year throughout my retirement period can I expect to receive*? Using the theory of compound interest and the probability theory I made my pension plan using several already developed mathematical models. Furthermore, I had to make assumptions based on the market data and demographic statistics regarding the interest rate, expected life span and the discount rate. The final result or the answer that I have obtained is that I can expect almost €4,000 of supplementary pension per year. I believe that this essay will contribute to the debate on the standard of living in old age, especially among young people and, consequently, it will raise our awareness of the need to save money for old age.

Word count: 197

TABLE OF CONTENTS

[1. Introduction 5](#_Toc469214302)

[***1.1 Macroeconomic background of the problem*** 5](#_Toc469214303)

[***1.2*** ***Approaching the microeconomic problem*** 6](#_Toc469214304)

[2. Capital in my savings account at the time of retirement 8](#_Toc469214305)

[***2.1*** ***The theory of compound interest – definitions and derivations*** 8](#_Toc469214306)

[*2.1.1* *Interest rate* 8](#_Toc469214307)

[*2.1.2* *Present value* 8](#_Toc469214308)

[***2.2*** ***Application of the theory – the capital at the end of my saving period*** 9](#_Toc469214309)

[*2.2.1* *Derivation of accumulated value of my contributions* 9](#_Toc469214310)

[*2.2.2* *Market research on interest rates* 10](#_Toc469214311)

[*2.2.3* *Calculation of capital for my case* 11](#_Toc469214312)

[3. The whole life annuity which I can expect in retirement 12](#_Toc469214313)

[***3.1 The probability theory – definitions and derivations*** 12](#_Toc469214314)

[*3.1.1* *The future lifetime of a person aged x* 12](#_Toc469214315)

[*3.1.2* *The curtate future lifetime* 14](#_Toc469214316)

[***3.2*** ***Application of the theory – whole life annuity*** 14](#_Toc469214317)

[*3.2.1* *Derivation of a whole life annuity* 14](#_Toc469214318)

[*3.2.2* *Market research on discount rates* 16](#_Toc469214319)

[*3.2.3* *Calculation of annuities for my case* 16](#_Toc469214320)

[4. Discussion and conclusion 18](#_Toc469214321)

[Bibliography 19](#_Toc469214322)

[Appendix 1: Slovenia 2010 reference population mortality table SCO65 20](#_Toc469214323)

[Appendix 2: Screenshot of an excel spreadsheet 22](#_Toc469214324)

**TABLE OF FIGURES**

[Figure 1: Population pyramids for Slovenia, 2011 and 2060 5](#_Toc469214325)

[Figure 2: Time line – cash flow 9](#_Toc469214326)

[Figure 3: Pension plan annual yields achieved in the period from 2011 to 2015 (in %) 11](#_Toc469214327)

[Figure 4: Time line – cash flow of annuities 15](#_Toc469214328)

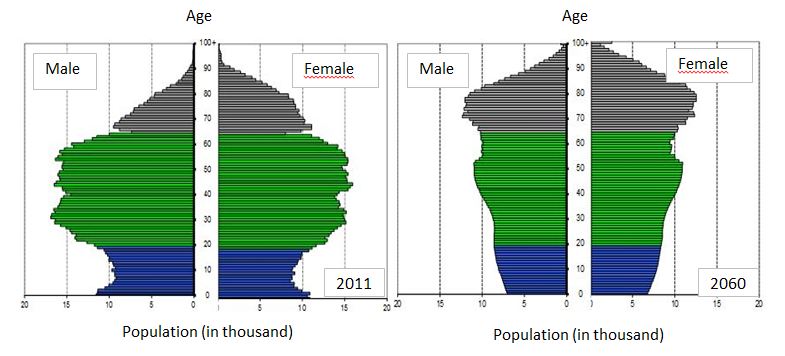
[Figure 5: Risk free term structure on 3rd of August 2016 16](#_Toc469214329)

# Introduction

## **1.1 Macroeconomic background of the problem**

The population ageing in Europe is one of the most famous phenomena nowadays. The same applies for my home country Slovenia. The problem of population ageing in Slovenia together with predictions for the future was thoroughly presented by Čok, Sambt and Majcen, famous Slovenian economists, in 2010[[1]](#footnote-1). By comparing the two population pyramids in the Figure 1 (for years 2011 and 2060), one can notice the narrowing of the figures between the years. To be more specific, in the first pyramid there is a huge generation of workers (green colour) and a small number of retired persons (grey colour). In the second one, there are more retired people and fewer employed persons then in 2011. Figure 1 shows the extending of the life span – the major reason for the strain on the pension system.

Figure 1: Population pyramids for Slovenia, 2011 and 2060



Source: Slovenian Government office for development and European cohesion policy. *Change of pension system.* 2011*.*

Ageing Slovenian population increases the strain on the pension system. Thus, in scientific literature scholars frequently discuss the problem of sustainability of public pension system based on the scheme Pay as you go (PAYG), which is used in the majority of European countries. However, in Slovenia PAYG represents the first pillar of pension system[[2]](#footnote-2). The countries with the greatest problems regarding public pension systems are France, Spain and Italy[[3]](#footnote-3). Likewise, in Slovenia in 2014 the ratio between employed and retired people was approximately 1:1.3. In practice this means that for provision of pensions, every employed person has to work for 1.3 retired persons[[4]](#footnote-4).

The PAYG scheme is based on intergenerational redistribution by which the payed contributions of present generation of workers are promptly spend for disbursing pensions of present generation of retired people. That is how the PAYG scheme provides society a minimal social stability[[5]](#footnote-5).

In contrast to redistribution of income across generations (from young to old), there is only low intragenerational redistribution (from rich to poor). In Slovenia, contributions are proportional to earnings (i.e. the Bismarck system)[[6]](#footnote-6). However, it is quite obvious that people cannot completely rely on this system because pensions are going to become lower due to the lack of financial resources.

Besides the public pension system, there is also the private one; in Slovenia this is the second and the third pillar of the pension system. In its framework assets are managed privately and financing is mainly in the form of funded scheme. Due to the instability of the public pension system, the private one has become used more widely[[7]](#footnote-7).

## **Approaching the microeconomic problem**

The private pension system is the basis for solving the specific microeconomic problem, which I pose in my extended essay:

**If I put a specific amount of money in euros into a savings account each year while I am working, how much additional money per year throughout my retirement period can I expect to receive?**

To solve this problem I have to make assumptions based on market data and demographic statistics regarding the interest rate, expected life span and discount rate.

The main reason I chose this problem is that the International Baccalaureate Diploma Programme of mathematics in my school does not cover financial and actuarial mathematics. Therefore, this essay gives me an opportunity to broaden my knowledge in these fields. Additionally, I believe that in the future, when I start saving for my retirement, I will be able to use my knowledge in practice.

In my essay I am going to develop a mathematical strategy for solving the above stated problem using financial mathematics and probability theory. The essay presents an individual perspective and application of furtherly discussed mathematical knowledge in practice.

I am going to approach the problem in two steps:

Step 1: Calculation of the final sum of money in my savings account at the time of retirement (Chapter 2.2) using theory of compound interest (described in Chapter 2.1).

Step 2: Calculation of the annuities which I can expect in retirement period (Chapter 3.2) using probability theory (Chapter 3.1).

# Capital in my savings account at the time of retirement

## ***The theory of compound interest – definitions and derivations***

Reviewing the literature on financial mathematics in order to solve the stated problem we have to learn more about the theory of compound interest. In this chapter the basic concepts of the theory of compound interest, particularly interest rates and present value are presented.

### *Interest rate*

Starting with the definition, interest rate is a price for which lender lends money to borrower for a certain time unit, typically one year[[8]](#footnote-8). It is expressed as a percentage of money lent, denoted by . If the price is expressed in the amount of money it’s called interest.

According to the theory of compound interest the interest itself earns interest[[9]](#footnote-9), meaning, that after each compounding period (i.e. the time interval at the end of which interest is added to the capital) the interest is not paid out but added to the capital and earns new interest in the next compounding period.

Therefore, interest rate is always stated in conjunction with time unit and compounding period[[10]](#footnote-10). If these two are identical the interest rate is called effective interest rate. In this case, interest is compound at the end of time unit[[11]](#footnote-11).

If we use annual effective interest rate, where interest is compound at the end of the year, the accumulated value of invested money after one year is equal to , where is called accumulation factor. Consequently, is the accumulated value after years.

### *Present value*

Determining a present value, i.e. today value of future cash flows, is a reverse operation of determining an accumulated value. Accumulated value after years is[[12]](#footnote-13):

To get a present value[[13]](#footnote-14) we divide the identity with (we assume that interest rate is positive, therefore ):

To simplify the identity we denote and obtain[[14]](#footnote-15):

The factor is called the discount rate and discounting is simply the reverse of compounding[[15]](#footnote-16).

## ***Application of the theory – the capital at the end of my saving period***

In this chapter we develop a mathematical model on the basis of the previously presented theory of compound interest, by which we can calculate the sum of money that I will have in a savings account at the end of the saving period, i.e. on the date when I retire. Let’s call it the capital at the end of the n–th year . For determining appropriate interest rate we are going to check current interest rates for savings offered by Slovenian pension companies, insurance companies and banks.

### *Derivation of accumulated value of my contributions*

Let annual effective interest rate remains constant during the saving period. Furthermore, we assume that at the end of every year during the saving period I contribute in a savings account the additional amount of money . And, we are looking for the value of my capital at the end of the saving period .

My annual contributions and the value of my capital at the end of -th year are illustrated on the following time line in Figure 2.

Figure 2: Time line – cash flow[[16]](#footnote-17)

~~. . .~~

~~. . .~~

~~. . .~~

*. . .*

*. . .*

*. .*

*. .*

*…*

*…*

Retirement

date

*Retirement*

*date*

Starting date

of work

At the end of the first year the accumulated value is equal to the first contribution :

At the end of the second year the accumulated value is equal to the first contribution multiplied with accumulation factor and the second contribution :

Similarly, the accumulated value at the end of the third year can be written as follows:

The accumulated value at the end of the year can be expressed with the following identity:

Let say that all the contributions equal to constant ; . Now, the value of my capital at the end of -th year is:

One can notice, that on the right side of identity the quotient of the adjacent terms (e.g. the first term divided by the second term) always equals to . It means that the terms on the right side of the identity represent geometric sequence.

Sum of the terms in geometric sequence[[17]](#footnote-18) expresses the value of my capital at the end of -th year by contributions and annual effective interest rate :

### *Market research on interest rates*

In Slovenia, there are one bank, five insurance companies and three pension companies offering pension plans. This is a form of a saving plan with guaranteed minimum annual yield. The annual yield (expressed in percentage) is the effective annual interest rate in calculation of accumulated value. Figure 3 shows annual yields achieved by different pension plans for the last 5 years.

Figure 3: Pension plan annual yields achieved in the period from 2011 to 2015 (in %)[[18]](#footnote-19)

Source: Own elaboration based on data from *Annual reports of Slovenian pension and insurance companies and Reports by The Securities Market Agency*. 2011 – 2015.

Yields achieved by different pension funds vary a lot. In the year 2015 yields were between 1.6% and 4.5%. For our calculation we choose the average, i.e. 2.8%. Moreover, we should be aware of the fact that financial markets are very dynamic, and therefore we should be aware of their instability which could cause some uncertainty of results.

### *Calculation of capital for my case*

Let’s assume that my salary will allow me to save €100 every month. This will enable me to invest €1.200 every year. Using the effective interest rate chosen in the previous section, the value of my capital at the end of the 40-th year of my length of service is equal to[[19]](#footnote-20):

# The whole life annuity which I can expect in retirement

In the previous chapter we have calculated the value of my capital at the end of my length of service. My plan is to use this capital for financing annuities, which I will receive annually after retirement. In this chapter we derivate a mathematical model to determine the value of my whole life annuity. To solve this problem, we had to examine the literature from the field of actuarial science, primarily the probability theory.

## ***3.1 The probability theory – definitions and derivations***

It is generally known that the amount of annuity depends on life expectancy, which can be described by a random variable defined and derived in my school workbook. Also, the term of conditional probability and the Bayes Theorem are introduced there[[20]](#footnote-21). It is important to be familiar with both to understand further concepts used in this essay.

### *The future lifetime of a person aged x*

The future lifetime of a person aged is a continuous random variable denoted by , which is continuously distributed on a domain where with a cumulative distribution function[[21]](#footnote-22):

The function represents the probability that a person aged will die within years, for any fixed . In actuarial notation the function reads as follows[[22]](#footnote-23):

In fact, this distribution function presents mortality rates, which are for Slovenian population published in Slovenian Annuity Tables SIA65[[23]](#footnote-24) (Appendix 1).

On the contrary, the probability that a person aged will survive at least years represents a survival function denoted by :

and actuarial notation is as follows[[24]](#footnote-26):

It is obviously that:

As mortality rates are given for one year, a connection between the probability that a person aged will survive at least years and the probability that a person aged will survive at least 1 year is used[[25]](#footnote-27). We follow Gerber and derivate this connection to make our further calculations easier.

The conditional probability that person aged will survive another years, after having attained the age equals[[26]](#footnote-28):

Using Bayes Theorem the conditional probability can be expressed with a bit more simple probability[[27]](#footnote-29):

To go further, the probability that a person aged will survive at least years is equal to the product of the probability that a person aged will survive at least years and the conditional probability that person aged will survive another years, while we already know that he will survive age [[28]](#footnote-30),[[29]](#footnote-31):

Finally, here is the corollary[[30]](#footnote-32) which we are looking for. The probability that a person will survive at least years equals the products of probabilities that a person will survive at least 1 year:

The corollary can be proven with mathematical induction.

### *The curtate future lifetime*

As I will receive my annuities annually, the future lifetime should be curtailed in a way that it presents only the integer part of a continuous random variable :

The square brackets denote the integer part. Discrete random variable represents the number of completed future years lived by person aged ; where x = . The probability distribution of the discrete random variable is given by[[31]](#footnote-35):

Assuming that is continuous in enables us to switch inequalities on the right side of identity and express probability with cumulative distribution function[[32]](#footnote-36):

From here Macdonald and Hardy derivate the following theorem: the probability that a person aged will live another years is equal to the probability that person aged will live another years multiplied with the probability that a person aged will die next year:

We prove this by using the survival function[[33]](#footnote-37).

## ***Application of the theory – whole life annuity***

In this chapter we develop a mathematical model on the basis of the previously presented theory, by which we can calculate the value of my whole life annuity.

### *Derivation of a whole life annuity*

If I die between ages and then a discrete random variable is equal to . As the annuities are payed at the end of the year, the last annuity will get my descendants. Figure 4 represents money that I will put into bank account and received annual annuities .

Figure 4: Time line – cash flow of annuities[[34]](#footnote-38)

. . .

~~. . .~~

~~. . .~~

~~. . .~~

~~. . .~~

~~. . .~~

~~. .~~

Retirement

date

Date of the death

Let’s say that after my retirement I will receive payments. To simplify the derivation we assume that each of payments equals 1. The present value of the annuity payments at the time of my retirement represents a random variable and it is equal to[[35]](#footnote-39):

Or shortly by using actuarial notation[[36]](#footnote-40):

Using the result from section 3.1.2 the probability distribution for a random variable is equal to[[37]](#footnote-41):

If we want to calculate an expected average outcome of a discrete random variable we have to multiply each value of random variable with its probability. Then, we have to summarise the products over all possible values of random variable [[38]](#footnote-42):

Actuaries denote it by :

### *Market research on discount rates*

European Insurance and Occupational Pensions Authority (EIOPA) publishes the relevant risk-free interest rate term structures which are applied by insurance companies to discount cash-flows of the insurance obligations. Risk free term structure is presented in Figure 5.

Figure 5: Risk free term structure on 3rd of August 2016

Source: *European Insurance and Occupational Pensions Authority.* 2016.

Let assume that I will retire after 45 years (5 years to finish education and 40 years of service; red vertical line in the graph) and that I will live another 20 years (black vertical line in the graph). In this period the risk free rate varies from 2,2% to 2,8%. We choose the average value (2,5%) as interest rate in calculation of present values. Yet, we have to be aware of the fact that in 45 years the risk free interest rate will be different and therefore the chosen value should be viewed with caution.

### *Calculation of annuities for my case*

Let me repeat my plan: using the saved capital I will finance the annuities during my retirement period. This means that the expected present value of annuities at the retirement date () has to be equal to the capital saved at the end of my length of service (after 40 years of saving):

As we calculated in Section 2.2.3, the capital saved at the end of my length of service when is equal to . If we want to calculate the annuity , we have to make equal to the expected present value of annuities at the end of length of service:

Using interest rate the discount value which was defined in Section 2.1.2 is equal to:

My annuity is then[[39]](#footnote-43):

According to the result obtained I can expect almost €4,000 of supplementary pension per year.

# Discussion and conclusion

In the present essay I have sought to answer the research question: If I put €1.200 into a savings account each year while I am working, how much additional money can I expect to receive throughout my retirement period? Using the theory of compound interest and the probability theory, I developed a mathematical model and found that I may expect almost €4,000 of supplementary pension per year.

While I was working on my extended essay I learned many new mathematical concepts which I then linked with my existing knowledge gained in school. I applied all of the concepts in practice, on my personal example. Furthermore, I will be able to apply my newly gained knowledge when I get job and start saving.

Although this essay was carefully prepared, I am aware of its limitations. Firstly, situation on financial markets is highly unpredictable. The model does not take into account neither future market recovery nor the prolonged current low interest rate environment. Therefore, my assumptions and the result obtained have to be considered with this instability in mind.

Secondly, there are some other limitations in my essay that could be taken as an opportunity for further research. For example, various investment strategies which would lead to alternative, more realistic assumption regarding interest rate might be explored. Also, one could make their own life tables. Unfortunately, I could not do this within the limits of my essay.

I believe that this essay contributes to better understanding of the used concepts by practical example and thorough research of mathematical background behind the formulae. I have also combined the already existing models and through this developed my own ones. However, due to the fact that the length of my essay is already reaching the upper limit, I could not include all proofs within the scope of the main text and they are therefore included in footnotes.

Last but not the least, I believe that this essay will contribute to the debate on standard of living in old age especially among young people and consequently, it will raise our awareness of the need to save money for old age.

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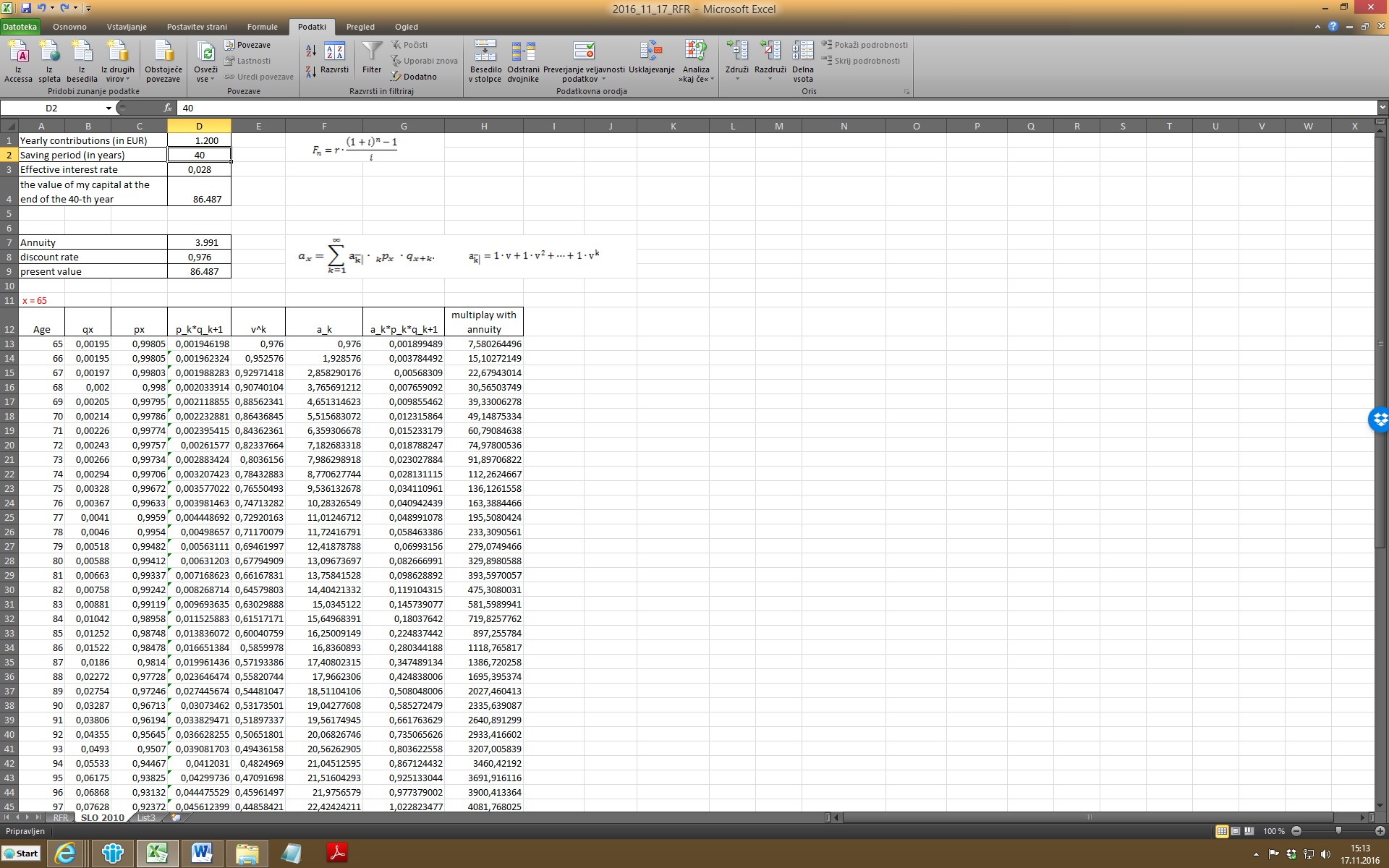
# Appendix 1: Slovenia 2010 reference population mortality table SCO65

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Age | SIA65 | SIA65 | Age | SIA65 | SIA65 | Age | SIA65 | SIA65 | Age | SIA65 | SIA65 |
| Men q\_x | Woman q\_y | Men q\_x | Woman q\_y | Men q\_x | Woman q\_y | Men q\_x | Woman q\_y |
| 0 | 0.01842 | 0.01860 | 36 | 0.00083 | 0.00039 | 72 | 0.01238 | 0.00410 | 108 | 0.38475 | 0.35465 |
| 1 | 0.00149 | 0.00147 | 37 | 0.00089 | 0.00041 | 73 | 0.01345 | 0.00460 | 109 | 0.42006 | 0.37932 |
| 2 | 0.00070 | 0.00081 | 38 | 0.00093 | 0.00043 | 74 | 0.01455 | 0.00518 | 110 | 0.45697 | 0.40444 |
| 3 | 0.00049 | 0.00052 | 39 | 0.00095 | 0.00046 | 75 | 0.01571 | 0.00588 | 111 | 0.49536 | 0.42987 |
| 4 | 0.00042 | 0.00038 | 40 | 0.00097 | 0.00049 | 76 | 0.01701 | 0.00663 | 112 | 0.53512 | 0.45947 |
| 5 | 0.00035 | 0.00029 | 41 | 0.00099 | 0.00053 | 77 | 0.01857 | 0.00758 | 113 | 0.57609 | 0.50420 |
| 6 | 0.00027 | 0.00023 | 42 | 0.00104 | 0.00057 | 78 | 0.01989 | 0.00881 | 114 | 0.61810 | 0.55079 |
| 7 | 0.00021 | 0.00018 | 43 | 0.00110 | 0.00063 | 79 | 0.02172 | 0.01042 | 115 | 0.66098 | 0.59902 |
| 8 | 0.00017 | 0.00014 | 44 | 0.00119 | 0.00069 | 80 | 0.02424 | 0.01252 | 116 | 0.70451 | 0.64865 |
| 9 | 0.00016 | 0.00012 | 45 | 0.00129 | 0.00077 | 81 | 0.02764 | 0.01522 | 117 | 0.74847 | 0.69940 |
| 10 | 0.00018 | 0.00011 | 46 | 0.00140 | 0.00086 | 82 | 0.03213 | 0.01860 | 118 | 0.79262 | 0.75098 |
| 11 | 0.00020 | 0.00011 | 47 | 0.00153 | 0.00096 | 83 | 0.03782 | 0.02272 | 119 | 0.83671 | 0.80304 |
| 12 | 0.00022 | 0.00012 | 48 | 0.00166 | 0.00107 | 84 | 0.04471 | 0.02754 | 120 | 1.00000 | 1.00000 |
| 13 | 0.00025 | 0.00013 | 49 | 0.00180 | 0.00118 | 85 | 0.05270 | 0.03287 |  |  |  |
| 14 | 0.00029 | 0.00016 | 50 | 0.00198 | 0.00128 | 86 | 0.06160 | 0.03806 |  |  |  |
| 15 | 0.00035 | 0.00019 | 51 | 0.00220 | 0.00139 | 87 | 0.07108 | 0.04355 |  |  |  |
| 16 | 0.00042 | 0.00023 | 52 | 0.00245 | 0.00149 | 88 | 0.07958 | 0.04930 |  |  |  |
| 17 | 0.00048 | 0.00025 | 53 | 0.00271 | 0.00158 | 89 | 0.08797 | 0.05533 |  |  |  |
| 18 | 0.00052 | 0.00027 | 54 | 0.00299 | 0.00166 | 90 | 0.09627 | 0.06175 |  |  |  |
| 19 | 0.00056 | 0.00028 | 55 | 0.00328 | 0.00169 | 91 | 0.10464 | 0.06868 |  |  |  |
| 20 | 0.00059 | 0.00027 | 56 | 0.00358 | 0.00173 | 92 | 0.11334 | 0.07628 |  |  |  |
| 21 | 0.00062 | 0.00027 | 57 | 0.00391 | 0.00177 | 93 | 0.12271 | 0.08469 |  |  |  |
| 22 | 0.00064 | 0.00027 | 58 | 0.00431 | 0.00183 | 94 | 0.13305 | 0.09402 |  |  |  |
| 23 | 0.00065 | 0.00026 | 59 | 0.00477 | 0.00189 | 95 | 0.14981 | 0.11130 |  |  |  |
| 24 | 0.00064 | 0.00026 | 60 | 0.00531 | 0.00195 | 96 | 0.16356 | 0.12398 |  |  |  |
| 25 | 0.00063 | 0.00026 | 61 | 0.00590 | 0.00195 | 97 | 0.17813 | 0.13768 |  |  |  |
| 26 | 0.00061 | 0.00027 | 62 | 0.00652 | 0.00197 | 98 | 0.19349 | 0.15241 |  |  |  |
| 27 | 0.00060 | 0.00028 | 63 | 0.00717 | 0.00200 | 99 | 0.20965 | 0.16819 |  |  |  |
| 28 | 0.00058 | 0.00028 | 64 | 0.00753 | 0.00205 | 100 | 0.22657 | 0.18503 |  |  |  |
| 29 | 0.00056 | 0.00029 | 65 | 0.00779 | 0.00214 | 101 | 0.24424 | 0.20292 |  |  |  |
| 30 | 0.00055 | 0.00030 | 66 | 0.00816 | 0.00226 | 102 | 0.26262 | 0.22184 |  |  |  |
| 31 | 0.00055 | 0.00031 | 67 | 0.00859 | 0.00243 | 103 | 0.28167 | 0.24176 |  |  |  |
| 32 | 0.00057 | 0.00033 | 68 | 0.00913 | 0.00266 | 104 | 0.30132 | 0.26266 |  |  |  |
| 33 | 0.00061 | 0.00034 | 69 | 0.00979 | 0.00294 | 105 | 0.32153 | 0.28446 |  |  |  |
| 34 | 0.00068 | 0.00036 | 70 | 0.01055 | 0.00328 | 106 | 0.34223 | 0.30712 |  |  |  |
| 35 | 0.00076 | 0.00037 | 71 | 0.01138 | 0.00367 | 107 | 0.36333 | 0.33055 |  |  |  |

**Age shifts for population mortality – fundamental cohort 1965**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Birth year | Male | Female | Birth year | Male | Female | Birth year | Male | Female |
| up to1950 | 2 | 2 | 1976 | -2 | -2 | 2002 | -6 | -5 |
| 1951 | 2 | 2 | 1977 | -2 | -2 | 2003 | -6 | -5 |
| 1952 | 2 | 2 | 1978 | -3 | -2 | 2004 | -6 | -5 |
| 1953 | 2 | 1 | 1979 | -3 | -2 | 2005 | -6 | -6 |
| 1954 | 1 | 1 | 1980 | -3 | -3 | 2006 | -7 | -6 |
| 1955 | 1 | 1 | 1981 | -3 | -3 | 2007 | -7 | -6 |
| 1956 | 1 | 1 | 1982 | -3 | -3 | 2008 | -7 | -6 |
| 1957 | 1 | 1 | 1983 | -3 | -3 | 2009 | -7 | -6 |
| 1958 | 1 | 1 | 1984 | -4 | -3 | 2010 | -7 | -6 |
| 1959 | 1 | 0 | 1985 | -4 | -3 | 2011 | -7 | -6 |
| 1960 | 0 | 0 | 1986 | -4 | -3 | 2012 | -8 | -6 |
| 1961 | 0 | 0 | 1987 | -4 | -4 | 2013 | -8 | -6 |
| 1962 | 0 | 0 | 1988 | -4 | -4 | 2014 | -8 | -6 |
| 1963 | 0 | 0 | 1989 | -4 | -4 | 2015 | -8 | -6 |
| 1964 | 0 | 0 | 1990 | -4 | -4 | 2016 | -8 | -6 |
| 1965 | 0 | 0 | 1991 | -5 | -4 | 2017 | -8 | -7 |
| 1966 | -1 | -1 | 1992 | -5 | -4 | 2018 | -8 | -7 |
| 1967 | -1 | -1 | 1993 | -5 | -4 | 2019 | -8 | -7 |
| 1968 | -1 | -1 | 1994 | -5 | -4 | 2020 | -8 | -7 |
| 1969 | -1 | -1 | 1995 | -5 | -4 |
| 1970 | -1 | -1 | 1996 | -5 | -5 |
| 1971 | -2 | -1 | 1997 | -6 | -5 |
| 1972 | -2 | -2 | 1998 | -6 | -5 |
| 1973 | -2 | -2 | 1999 | -6 | -5 |
| 1974 | -2 | -2 | 2000 | -6 | -5 |
| 1975 | -2 | -2 | 2001 | -6 | -5 |

# Appendix 2: Screenshot of an excel spreadsheet



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16. Contributions paid on a saving account are shown on the upper side of the time line where subscripts represent years. On the underside of the time line there is the value of my capital at the end of -th year or the amount received from the savings account Fn. [↑](#footnote-ref-17)
17. The sum of the numbers representing geometric sequence is equal to . [↑](#footnote-ref-18)
18. The names of banks, insurance companies and pension companies are written in Slovene language. [↑](#footnote-ref-19)
19. The result is rounded up to the nearest euro. [↑](#footnote-ref-20)
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27. Proof: [↑](#footnote-ref-29)
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33. Proof: [↑](#footnote-ref-37)
34. Money paid into bank account Fn is shown on the upper side of the time line. On the underside of the time line there are annual annuities , where subscripts represent years.. [↑](#footnote-ref-38)
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39. The result is rounded up to the nearest euro. [↑](#footnote-ref-43)